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# Introduction to Computational Algebraic Geometry in Robotics

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## ABSTRACT

Computational algebraic geometry (CAG) offers powerful methods for solving complex problems in robotics, particularly in robot motion planning. Traditional numerical approaches in robotics are complemented and often outperformed by algebraic geometry techniques, which utilize polynomial equations to model and solve problems related to robot kinematics, dynamics, and path planning. This paper provides an introduction to CAG, highlighting its application in robotics, from basic concepts and techniques to advanced applications in robot perception and motion planning. We discuss key algebraic tools and their practical implications, offering insights into current trends and future directions in this interdisciplinary field.

Keywords: Computational Algebraic Geometry, Robot Motion Planning, Kinematics, Path Planning and Configuration Space

## INTRODUCTION

Computational algebraic geometry (CAG) is an area of mathematics that deals with solving systems of polynomial equations and understanding the properties of their solutions. Its application in robotics, especially in the domain of motion planning, has opened new avenues for efficient and robust solutions to complex problems [1-3]. This paper introduces the foundational concepts of CAG and explores its utility in robotic applications. Robot motion planning is a critical problem in robotics that involves determining a collision-free path for a robot to move from one position to another. Traditional methods often rely on enumerative strategies that become inefficient as the complexity of the robot increases [4-6]. Algebraic geometry provides a more efficient framework by representing the robot's configuration space with polynomial equations, enabling the use of geometric and topological properties to design better algorithms. The use of algebraic geometry in robotics can be traced back to the work of John Canny, who developed the Bernstein-Khovanskii-Kushnirenko algorithm to address robot kinematics problems. This approach, focusing on the topology of algebraic curves and the real solutions of polynomial systems, marked a significant shift from distance-based or metric-based methods [7-9].

## Introduction to Computational Algebraic Geometry

One classical problem in robot control that is quite well suited for investigation using CAG is the problem of robot motion planning. In this problem, one is required to determine how a robot arm, composed of several rigid links, could move from one end effector position to another avoiding obstacles [ArtcileId: 76fc561a-3be9-4b06-9ac9-c24ccaaa6a08]. This can be done by enumerating all possible joint angles, and testing to ensure that each of these configurations does not intersect with the obstacles [10-12]. However, this is an inefficient approach that scales exponentially with the number of links comprising the robot. A more efficient alternative is to use methods from CAG. In these methods, the robot configuration space is studied as the solutions of polynomial equations, and the topology and geometry of the solutions are then used to design a faster algorithm. It turns out that configuration spaces of simple robots, such as planar linkages (without free or hanging links) and robots with spherical joints and revolute joints can be described by collections of algebraic varieties [ArtcileId: 76fc561a-3be9-4b06-9ac9-c24ccaaa6a08] [13-16]. In the case of simply connected robots, configuration spaces are tori. Configuration spaces of the Complanar 3-RPR and 3-RPR mechanisms are computed using algebraic geometry and overturned results about their connectivity. In recent years, algebraic geometry, and in particular computational algebraic geometry (CAG), has been increasingly applied in robotics [ArtcileId: 0327ebeb-1009-49a7-86de-7f7eafe9e53d]. While traditional approaches to robotic problems have involved mostly numerical and linear algebra methods, algebraic geometry provides a new suite of tools that can complement and sometimes replace these methods. This can perhaps be traced back to the work of Canny, who first developed an algorithm (the "Bernstein-Khovanskii-Kushnirenko" algorithm) for solving a range of problems in robot kinematics, including those involving path planning [ArtcileId: bd4d9bcb-9b7f-4fd1-a4ed-6b058dcef2eb]. The method provided a new approach to these challenging problems that was based on the topology of algebraic curves and real solutions of polynomial systems, rather than on distance or other metric-based criteria [14-20].

## Basic Concepts and Techniques

The computation of real solutions of system equations and inequalities often takes a step with the help of quantifier elimination. When solving a system of polynomial equations, real and complex systems are classified, where complex zeros are roots of Göbner basis. If we solve a so-called "real and regular" system, the system of equations has only real solutions with a positive dimension in Eq lies instead we call SISO. Any zero must be a solution to the input equations (i.e. the so-called regular situation) [20-24]. In general, the number of real solutions (including redundancies) can be obtained by counting the number of non-zero real solutions of the pseudo-Boolean solution of the inequations in Eqres instead. We can get the western height by a projection approach of the 'Student approach' rather than by the classical formulations. If an equivalent system of functions, the projection and the resulting polynomials have a unique and bijective view in an approximative sense. When quantifier elimination is performed in this projection, the equivalence of the systems Say [25-28]. The removal of movement redundancy by taking these joint parameters approximate results in a preconditioned overdetermined system of trigonometric equations with a rational parametric polynomial structure giving rise to a variety of 2D solutions in the three-dimensional configuration space of the end-effectors when both constraint equations have a complete set of 7× 7 resultants given in [27]. We observe that the intersection line between implicitly and parametrically given surfaces called surface trace efficient seeking is made. Besides, it turned out that its parametric description has a bicubic polynomial structure. This is a highly efficient structure used by certain robotic applications like visibility and accessibility analysis, path evaluation for trajectory and knots, and guiding-curve parameterization showing the pricing of the most important key points from the motion planning point of view guiding curve [29-32]. The main characteristic of algebraic geometry in computational methods is solving systems of equations. More precisely to the dimension of the variety of solutions, the implicitization of parametric surfaces obtained as B´ezier or non-rational freefo¨rm surfaces, the removal of the base point, etc. One of the closed-related fields of robot kinematic analysis is also a part of this area. Both industrial and humanoid robots are multibodied systems constructed with hybrid joints. Those are mainly combined by using cylindrical and spherical joints. This hybrid joint characteristic can make kinematic analysis problems more complicated to solve [33-38].

## Applications in Robotics

It's also possible to abstract portions of robot end-effectors as immobile solid objects (loosely abstracting robot linkages as solid obstacles) [39-41]. It is natural to ask questions like, "Is it possible to move object A to location 1, and object B to location 2 while avoiding collisions and obeying coordination constraints?" A necessary condition satisfied to secure the possibility of a solution is that the associated configuration space (often including a range of examples of solid subspaces) be nonempty, i. e. the elements represent parameter values for which the meromorphic function has at least one zero. If two meromorphic functions share the same zeros, they are called separable. The vanishing of the resultant determines the condition that two separating equations must satisfy for a real separation to occur. Robots can be modeled by chair graphs, colored edge graphs, edge-labeled trees, or relative incident clusterings, and observations about their theoretical spaces can have impacts on robotics practice. In particular, semi-algebraic connectivity queries (as treated here) can impact robotics algorithms for the analysis of, e.g., kinematic workspace, configuration space, singular configuration, and manipulation planning [42-44]. Solvers and automation for problems in computational algebraic geometry can be employed in a wide variety of applications—problems that reduce in complexity to polynomial systems, or systems of equation, can be attacked with bespoke analogs of symbolic and numeric algorithms, implemented as computational tools and software libraries. The time and space costs associated with their deployment are large in all but very special cases—compared to, e. g. neural networks. However, advances in mathematical algorithm design, particularly during the 1990s and 2000s, enabled large complexity gains for a variety of inputs. In this article, adapted from the 2021 talk of the same name, I review current trends illustrating the utility of algebraic geometry to enable new methods for ancient and contemporary problems. The focus here is on problems related to robotics (for specialists: manipulation, including wheeled mobile, legged, underwater, and aerial platforms), in line with the workshop's thematic focus [45-48].

## Robot Kinematics and Dynamics

### Algebraic Geometry in Robot Motion Planning

Trajectory planning is a basic task in robotics. Cartesian motion planning consists of defining the movements of a manipulator by describing the trajectory a robot end-effector has to sweep to cover the desired path in the global workspace. Typical polynomial models used here are given in dedicated monographs. A famous model to study a mobile manipulator with 6-DOF (Degree Of Freedom) is given by the direct and inverse kinematic models, being algebraic ones. The object of the direct kinematic model is to solve the direct position problem, which is having a pose (position and orientation) of the end-effector to calculate the position of the arm joints. Its counterpart solution problem is the inverse position problem. One of its fundamental approach is Gröbner Bases method assisting a causal finding of the exact, algebraical solutions of the kinematics equations.

Algebraic geometry has a firm place in robotics. By its nature robotics is an interdisciplinary field which, among others, uses geometric methods to model robots and interpret their interaction with the surrounding world [49-51]. Computation is more and more dominating the state-of-the-art methods used in mobile robot motion planning. The exact, symbolical methods of reasoning about robot motion are based then on the algebraic geometry [52-54]. It enables the reasons based on the polynomial relation between geometric objects which, according to the Bézout theorem usually have a finite number of solutions specified over the complex projective space. In robotic motion planning at least the dimension zero solutions correspond to the possible configurations to ensure robot mobility.

### Configuration Space and C-Space Obstacles

C-space obstacles owe their name to the fact that they are situated in the C-space, the high-dimensional space in which the robot's configuration is represented. Different types of C-space models have been proposed including object-pose C-space, and manipulator C-space. Their removal from C-space is a key ingredient for the motion-planning model check, a critical aspect of the proposed approach. These obstacles are indeed extremely relevant in the context of robot motion planning in cluttered environments, and their computation is the key to the performance of model-check-based methods. The reduction of C-space dimensions and dimension configurations in motion planning has been a topic of interest and future work includes handling of these aspects. The removal of C-space obstacles plays a critical role in the performance of the model-check-based approaches introduced in this article [55- 57].

In robotics, it is interesting to employ geometric techniques that offer good performance both in terms of feasibility and optimality. Such techniques often rely on the construction of the so-called configuration space (C-space for short), a topological representation of the space of robot poses where collision situations are encoded. To accomplish this construction, it is necessary to identify the C-space obstacles where the robot can't move without colliding with the object. Configuration-space (or C-Space) obstacles are obstacles that are impossible to place within the robot's environment such that it is free to move, regardless of how it is positioned. A large number of model-check-based approaches to robotics motion planning build upon configuration space, which encodes all possible object configurations in an N-dimensional space [58-60].

### Algebraic Methods for Path Planning

Algebraic methods for path planning now start from the robot's kinematic model and the collision constraints. They all aim to obtain, in the most general case, the algebraic varieties—implicitly described by polynomial equations—of the enveloping constraints of the robot and obstacles. Although explicit polynomial representation of these objects is not demanded, an important task is to provide them in some form. Implicitization is always part of the solution although it might not be the dominant part of the computational complexity. To describe the robot's structure the Denavit–Hartenberg parameters could be used. Perception-driven methods make use of pre-built environment maps, the determination of useful landmarks, and plan graphs described as nodes and edges. Geographic landmarks, particularly triple junctions, could be chosen to build planar planar graphs in a deterministic manner. Rare or clustered occurrences of loosely defined special locations could be resolved by partitions of the floor in footprints as geometric markers. Note that concepts from algebraic- and differential geometry can be used to improve the sparse objects obtained [61-63]. To incorporate the structure of the environment to curb the graph size, several recent methods treat plan graphs with landmarks in light of a Minkowski addition of the robot and its free space. The shadowing information computed for the robot's bounding ball is augmented or programmed in the obstacle to modify some undetermined features of the landmark–obstacle projections which eventually affect the touch points with the obstacles. While the introduction of the robot's shape to the planning problem allows to use of informed edges and vertices, the pure focus on asymptotic optimal graphs and lattices is well-motivated given the increasingly powerful multi-query variant of search algorithms, offering informative directions: the Fiber Bundle describes all paths of the robot. While this appropriately models the information available on the robot's self-motion, the mean geodesic complete length and the best-fitting least-squares line segments hierarchically sub-stratify the fiber bundle. The work is relevant if the different geometric abstractions are automatically established at the FCL collision library preconditioning event or are discovered on the way [64-66].

Robotic path planning is the problem of setting a sequence of robot joint configurations (the path) to transport the robot's end effector from an initial to a final configuration while avoiding collisions with obstacles. The broader challenge of motion planning for dynamic environments coupled with a path planner is, in general, an open problem in robotics. In static scenarios, the robot is assumed to be in isolation. The key to the method's success is the use of graphs to iteratively prune "obvious" solutions that do not end in collision-free paths, guided by the result of kinematic feasibility tests and the sparsity of the resulting graph [67-70].

## Algebraic Geometry in Robot Perception

In the case of machine vision, the manifold (or the locus of forbidden moves) is easy to describe as a variety of a fixed degree. Typically given the observed pixels y, it is difficult to obtain the rays x that go through a fixed optical center and verify that there is a point at the rays with the same coordinates in say mm. As this path can manifest as a point, it illustrates another case of, seeing some parametric equations, going back to the Cartesian equation of the path. If a neural model learned the formula of Cp, it could thus learn to go back to the Cartesian point x where, provided the observed pixel is an image, the observed light path $Cx \in M$ of hitting the camera lens intersects the desired place x. It would result in a uniform subnetwork those uniform subnetworks are collapsed in the drawing and sometimes form low-dimensional manifolds [71, 72].

## Computational algebraic geometry of linear manifolds

Computational algebraic geometry has long offered possibilities for robot perception and possible control. In particular, it exploits the restriction on moves that are due to mechanics in robotics. Several positional restrictions are classically modeled using polynomial equations. For instance, a planar joint is usually modeled with a 1-D equation in the coordinates of the different parts. Moreover, in this setting, a plurality of neuron models for the paths of the motors has been suggested, with the hope that neural learning can grasp better the low dimensional structure of data of the robot than learning on the raw data [73, 74].

## Point Cloud Processing

New developments in edge infrastructure and 5G networks facilitate robotics within cloud-fog-edge computing environments. Such robotic applications require distributed 3D reconstruction, visual SLAM, and robot activity monitoring. This work introduces an edge-fog cloud architecture for reconstructing, in real-time, a 3D representation of an environment featuring sub-millimeter accuracy. A Fog Node has a Pointer machine to relay frames of RGBD cameras to the Cloud for reconstructing 3D models of gas and steam power plants. Fog devices aboard an unmanned ground vehicle employ the off-the-shelf RGB-D camera(s) to perform visual SLAM in real time. The position of the robot and maps of the environments are continuously communicated over 5G to the cloud, using a dedicated GNSS-based system for localization. Parameters estimated by a Structure Plus Motion (SfM) algorithm are calibrated in real-time, in the fog layer, by using pose measurements of fiducials. The accuracy of 3D reconstructions with and without real-time calibration is evaluated [75, 76]. Point cloud processing plays a vital role in modern robotics. Messages are acquired in the form of a point cloud and have to be converted to a 3D form for processing. The process of point map construction from the point cloud and depth registration presents one type of point cloud processing. To note the correct flow speed, maintaining reliability is highly important, while multiple kets consume major processing time. The Original Equipment Manufacturers (OEMs) recommend certain pumping systems to secure system warranties [77, 78]. Therefore, very little flexibility is available to apply modifications in securing security. The other is distributed atmospheric concentration measurements, usually much less dense than down-looking profile measurements, for which the authors look to adapt methods for radiosonde observations requiring complete atmosphere profiles when using Atmospheric Emitted Radiance Interferometers (AERIs) at Atmospheric Radiation Measurement (ARM) Program sites [79, 80]. As comprehensive profile measurements are desirable, the cloud height, pressure, and temperature measurements are quantified for AERIs, Microwave Radiometers (MWRs), and radiosondes.

## Object Recognition

The input of a mesh for object recognition given an RGBD image is usually a 3D range-only mesh. After the recognition process, the alignment of the obtained object pose with the original pose of the model should be done. If a closer object pose is needed, it can be achieved by optimization block composed of coherence point-to-point metric and alternating closest point ICP and exhaustively applying from different initial transformations. Our pipeline shows that the robot can autonomously recognize objects on a shelf locate the sensor within the object surface facing the user and use it for further interaction [81, 82]. We verify that the latest version of hvad 1.0.5 used for the work adds partial cloud integration. Together, both PCL and Open CV contain a variety of object recognition and descriptor functions. The basic object recognition solution is to search for features in a scene and try to find similarities between descriptors on a training model. For object recognition, Open CV allows the user to select different algorithms to describe the key points and search for matches in compound images. Scale-Invariant Feature Transform (SIFT) or Speeded Robust Feature (SURF) is available to be used either as feature detection and descriptor extraction or fast corner detection. A brute-force matcher can be tuned after the key descriptors are created, for example using Knn search. The basic object recognition pipeline follows the stages of descriptor modeling for 3D printing, the use of trained object descriptors for object recognition, and iterative closest point patch working matching to refine the detected object pose [83, 84]. After the alignment step has been performed, a different pipeline can be run to obtain an object model for recognition. By using normals, a segmentation of the object can be obtained via PCL by computing the difference of angles between adjacent points and clustering points into different segments [85, 86].

## Future Directions and Research Challenges

One of the seemingly infinite branches of the implementation of the just-described theoretical tools in robotics is the use of algebraic methods in algorithmic differentiation applied to motion optimization. The most studied methods are the dual number methods but their generalization to an algebraic setting is an open problem. Another less immediate application for robotic kinematics is the extension of the so-called genetic geometric mix approach. The idea of this approach is to consider a part of the pedals associated with a discrete path on acquired data to find a segment in the classical setting. The first question is to find efficient strategies to represent the part of the complete pedals. The second question is to decide what the algebra on them is. The complexity of converting with rational coefficients and the use of a bed basis seems a natural choice [87, 88]. As just mentioned, the idea of completion to augment varieties in algebraic geometry is an old tool. Essentially, this amounts to adding additional variables that will be perturbed to make the original problem "completely" decomposed. If these coordinates are chosen in a complex way, they will be able to represent a degree much lower than that of the classical construction. This area is still very young but is highly promising. One potential application in robotics is to use spectral numbers to give a good approximation of the singularity of a workspace using a few parameters. Another is to try to rearrange the closet part of it concerning the undetermined part to have equations that are degrees less important. The ground field for these applications could be computed based on the application, without needing to have a complexification such as the one used for classical Groebner basis algorithms. As witnessed in the previous sections, there are very high potential and recent applications of computational algebraic geometry as a tool in robot kinematics, robot learning, and generally in geometric problems in robot study, despite its heavy mathematical baggage. The following list of potential future improvements reflects largely in the geometric contexts just presented. Algebraic methods have huge potential for many other problems in robotics [89, 90]. Among them, movement synthesis and optimization would benefit greatly from geometric methods to deal with the capacities and configuration spaces of robots.

## CONCLUSION

Computational algebraic geometry provides a robust framework for addressing complex problems in robotics. By leveraging polynomial equations and geometric methods, it offers efficient solutions for motion planning, kinematics, and path planning. As research in this field continues to evolve, we can expect further advancements that will enhance the capabilities and performance of robotic systems.

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