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Mathematical Optimization Techniques in Sustainable Energy Systems Engineering

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ABSTRACT

This paper highlights the forefront of mathematical optimization techniques within sustainable energy systems engineering, encompassing both theoretical innovations and practical applications. The focus extends to key components such as microgrids, power systems, and energy carriers, integrating decision-making processes crucial for optimal investment, operational, and maintenance strategies. By leveraging multi-objective and many-objective frameworks, this issue addresses the critical balance between computational effort and solution quality, tailoring solutions to decision-makers preferences. As global energy systems face transformative pressures due to environmental concerns, this issue underscores the role of mathematical optimization in advancing the efficiency and sustainability of energy infrastructures. The discussion spans a variety of methodologies, including linear programming, mixed-integer programming, and robust optimization, with applications in renewable energy systems siting, sizing, and integration.

Keywords: Mathematical, Optimization, Techniques and Sustainable Energy

INTRODUCTION

The global energy landscape is undergoing unprecedented changes driven by the urgent need to address air pollution, climate change, and the rapid depletion of finite resources. These pressures necessitate the development and optimization of sustainable energy systems [1-4]]. This special issue is devoted to presenting the latest advancements in mathematical optimization techniques as they pertain to sustainable energy systems engineering, focusing on both theoretical advancements and practical implementations.

Mathematical optimization is indispensable in the decision-making processes that underpin the development of optimal investment, operational, and maintenance strategies for sustainable energy systems [5-7]. This issue seeks to balance computational effort with solution quality by integrating multi-objective and many-objective frameworks, ensuring that the solutions are aligned with decision-makers preferences. The rising global energy demand has exacerbated environmental issues due to the heavy reliance on non-renewable energy sources, leading to significant releases of pollutants. Sustainable energy resources, with their vast potential and minimal environmental impact, are crucial for addressing this demand [8-10]. However, the inconsistent and unpredictable nature of renewable energy sources such as solar and wind presents considerable challenges for their widespread adoption and integration into existing energy systems [11, 12]. This paper evaluates the fundamentals of mathematical optimization, including linear programming and its diverse applications in sustainable energy systems engineering. It explores state-of-the-art optimization techniques and their applications in the optimal siting, sizing, and integration of renewable energy systems. Additionally, we address the critical challenge of incorporating uncertainty into optimization models, which is essential given the inherent variability in renewable energy generation.

Sustainable Energy Systems Engineering

This paper is devoted to presenting recent developments in mathematical optimization techniques in the sustainable energy system engineering field. The aim is to include both theoretical methodologies and practical applications dealing with key optimization and energy conversion units like microgrids, power systems, energy carriers, and more. Decision–making processes are often included in different models to produce optimal investment, operational, or maintenance plans [Development of open-source instruments for in-situ measurements of waves in ice]. Hence, the models include several steps and tools for analyzing problems such as code-bloat [A Nationwide Multi-Location Multi-Resource Stochastic Programming Based Energy Planning Framework] [13-15]. Moreover, different methodologies are required to achieve the ratio between computational effort and the quality of the solution. To include the two main characteristics of the models, many authors turn attention to using the new multi-objective frameworks, or the so-called many-objective frameworks, mainly tailored for identifying the best trade-off and also the trade-off that fits the then decision-maker preference. All around the world, the energy sector is undergoing enormous amounts of changes. These changes stem from concerns about air pollution, climate change, and global warming. Many measures have been proposed to alleviate the rapid consumption of finite resources and energy security matters [A Nationwide Multi-Location Multi-Resource Stochastic Programming Based Energy Planning Framework] [16-18]. This includes the development of renewable energy sources, energy efficiency, and energy conservation. The development of any renewable or non-renewable resources for sustainable development requires a few technical, economical, and environmental constraints [Development of open-source instruments for in-situ measurements of waves in ice]. Furthermore, the development of economic and efficient new technologies for sustainable well-being is the main research work. Mathematical optimization plays a crucial role in decision-making processes to respond to the mentioned energy challenges [Multi-Objective Constrained Optimization for Energy Applications via Tree Ensembles] [19-20].

Overview of Sustainable Energy Sources

However, recent statistics indicate that the energy demand all over the world has caused acute problems due to the immense release of environmental pollutants because of the combustion of non-renewable energy resources [21-23]. To balance these environmental and energy demand requirements, many researchers and industrial societies have focused on sustainable energy resources that are available in the world in infinite quantities. It has a huge potential to produce electrical energy and the environmental disadvantages are substantially less as compared to traditional energy resources and other renewable energy resources such as solar energy, wind energy, and geothermal energy. The unpredictable, inconsistent, and less available natures of renewable energies such as solar, wind, geothermal, and hydro energy, to name but a few, have made small renewable energy utilization a major issue. This issue has led to the low adoption of renewable energy technologies in comparison to conventional energy resources [24-26]. The global environmental factors and sustained energy supply have transformed renewable energy techniques into one of the most hyped topics. Despite the immense breakthroughs and enormous capacity, renewable energy sources such as solar, wind, hydro, and geothermal energy are employed at a less-than-desired level compared to the impatient energy demand and the required environmental regulations [27-29]. Additionally, the less available and variable features of these sources make it even harder to switch renewable energy sources for non-renewable energy utilization in energy systems, particularly electrical energy systems. Non-renewable energy resources are utilized in power plants to produce electrical energy and are quite attractive primarily because of the availability of these resources [30, 31].

Fundamentals of Mathematical Optimization

To operate, optimize, or physically expand such systems, sophisticated engineering optimization techniques and powerful computational tools are required. Among various methodologies, mathematical optimization has been proven effective in handling complex decision-making processes using mathematical techniques. Generally, mathematical optimization models consider explicit mathematical objectives and define the decision variables and constraints explicitly. Selected decision variables will be optimized so that the declared objective function can be minimized or maximized [32-34]. Mathematical optimization turns out to be a strong tool for solving diverse problems from different engineering practices including sustainable energy systems. It selects from several inputs to provide solutions, such as input costs and production scale. The massive rise in the production and consumption of energy has led to increased greenhouse gas emissions and environmental degradation, which has negatively affected human health [35-37]. However, the increasing awareness of global warming, air pollution, and the rapid exhaustion of fossil fuels has motivated people to explore alternative solutions to meet the rising global energy demand [6]. Renewable energy sources are highly attractive solutions since they emit less CO2, have little to no environmental pollution, and are widely available. This has led to the development and expansion of technologies that focus on more sustainable energy systems, which include solar photovoltaic (PV), wind, biomass, geothermal, hydro, and many more [7, 38, 39].

Linear Programming

The Linear Programming (LP) model can be expressed as a standard form, where the optimization is done through state space methods for obtaining optimal controller gains. State space methods are formulated based on the discretized approximate model. LP techniques are used for optimizing problems in an application with large-scale and complex problems [40, 41]. Time-varying input of HCNG using different LP techniques such as dual method, dual simplex method, iterative method, and variational inequality technique are fruitful to test for reducing the total time while variations are applied to demand and system electricity costs.

Production costs can be formulated as a linear-convex function of the operational rates of CHP units [9]. The output of CHP and boiler hierarchical operation strategy is formulated as LP to minimize the operational production costs. Electric boilers and heat pumps can also be included in LP formulation. In energy systems engineering, LP is widely used and efficient for production capacity optimization or to determine the optimal operation strategy of the power-generation or heat-generating systems (for example district heating systems or gas turbine-based power plants) [10]. LP can also be advantageous in combined heat and power (CHP)-Production systems. Mathematical optimization techniques have become essential tools in the design and planning of energy systems. Linear programming (LP) is used to solve optimization problems where both the objective function and the constraints are linear [12]. LP is lightweight compared to other optimization techniques (e.g., mixed-integer programming, non-linear programming, etc.) and can produce solutions quickly. Linear programming aims to optimize the problem and to find the maximum or the minimum of the objective function [42].

Applications of Optimization Techniques in Sustainable Energy Systems Engineering

The traditional optimization has been widely applied in various introduced applications and it can be solved by using classical optimization techniques, such as mixed-integer linear programming [13]. However, there are increasing challenges in several optimal research areas, such as the need to handle more complex components involving renewable energy systems, the increasing need to consider the whole period of present-paper demand, and managing the operational assets and their guarantees to be able to model the stochastic behaviour of the renewable data accurately and efficiently, the requirement of reverting to more closest approaches in this context due to the nonlinear and stochastic components of renewable [43-45]. Due to these difficulties, models originated containing both classes of the lower and upper components. These models are however designed to reflect the complex relationship synthetically across the leader and follower, leading to a tri-level (TL) decision-making problem. This occurs particularly in a renewable energy system (RES) in which renewable power includes the F capacity and instantaneous electricity consumption represents the L capacity (i.e. the storage system) [46, 47]. Therefore, the RE leading sectors, stationary storage systems (SSS), and EVCs are influential for renewable investment and operation, including their operational profile trend and their capacity alternatives for their long-term changes. As a complementary to the existing work, in this section, we present a brief review of the state-of-the-art techniques in mathematical optimization and their applications in sustainable energy engineering. In particular, we will focus on the application of mixed-integer linear programming, mixed-integer quadratic programming, mixed-integer non-linear programming, multi-objective optimization, bi level, and tri-level optimization in the related problems [48, 49]. Moreover, we also introduce a data-driven approach for black-box optimization or mixed-integer linear programming by using supervised learning and tree ensemble [10].

Applications of mathematical optimization techniques in sustainable energy systems engineering are very common and penultimate. It can apply to problems such as power system scheduling and operation, transmission expansion planning, distribution planning, renewable energy integration, and energy storage scheduling [50, 51]. Moreover, optimization algorithms can also be used in the design of sustainable energy systems which are based on renewable resources. Existing literature reviews in this field have mainly focused on the application of classical optimization techniques such as (mixed-integer) linear programming, or meta-heuristic algorithms like genetic algorithms, particle swarm optimization, and gravitational search algorithms for sustainable energy systems [14].

Optimal Siting and Sizing of Renewable Energy Systems

Recent research efforts in sustainable energy systems are directed towards the development of large-scale power system models that simultaneously integrate supply-, demand- and storage-side options that are required to ensure system flexibility, resilience, and security without unintended increases in costs. An electric grid more reliant on dispatchable renewable energy sources, more controllable resources such as flexible demand and electric storage, and grid infrastructure modernized via high-voltage transmission, intra-hourly operation schemes, and demand-response programs to mitigate supply-and-demand imbalance, reduced energy-related greenhouse gas emissions and local air pollution are regarded [10, 52]. The codified boundaries, rules, assumptions, conceptions, and focal questions are absent in modern power grid operations. Traditional measures are not fully adapted to value grid services, services transactions, long-term resource or infrastructure siting, and energy-environmental performance properly. Mathematically, these complexities are non-convex and have heterogeneous attributes of combinatorial variables, continuous space, integer variables, and conditional values which need to be incorporated when global and local optimums are sought for the large models. Optimal siting and sizing of renewable energy systems is crucial for sustainable energy systems engineering [21, 53, 54]. With the rising demand for energy and the need for environmental protection, optimal management and control of available resources is a key issue. Factors such as the price of generated energy, greenhouse gas emissions, availability of resources, efficiency of energy conversion, natural resource requirement, and social impacts are considered in the comparison between renewable energy sources. Optimization tools and algorithms provide a suitable way to solve complex energy systems problems [17, 55, 56]. When applied to energy system planning, optimization techniques provide decisions regarding the whole system's behaviour based on knowledge of the mathematical description of the system and its components.

Challenges and Future Directions

However, both hyperparameter optimization for accurate and efficient modeling and scalability are the grand concerns in SES design problems. It may be anticipated that the rapid big data collection and AI-based modeling techniques will accelerate the replacement of the prescriptive models with the predictive ones that encourage big data analytics [57, 58]. A very serious concern about the pursuit of success of the AI-aided MO is the demand for the expert's wisdom, for the conciliatory converged solutions that make sense in the physical world. The future is bright with AI and MO contributing to achieving the global sustainability targets, and indeed the search for a mature technology is on. MO formulations in real-world SES engineering problems are influenced by plural factors, like i) non-linear relationships, ii) structure uncertainties, iii) diversity of energy sources, iv) intractable search space, v) frequent need for re-planning and timings, vi) conflicting objectives, and multifarious constraints. To capture the complexities: recent studies are convinced to acquire wisdom from other disciplines and to encourage further interdisciplinary research. A grand effort is now underway to wed domain knowledge, AI, and MO. Future R&D Opportunities [59, 60].

Challenges and Recent Emergent Solutions

CALLING experts: The various mathematical optimization (MO) formulations for sustainable energy systems (SESs) engineering problems epitomize the valiant efforts to advance several technologies [18]. This so humungous knowledge base aside, a few serious concerns have pushed the excited academic community to propose and investigate many attractive concepts and methodologies [20]. Though the SESs knowledge has tremendously increased, real wisdom and mature understanding of the MO are not available to the DA's and policy planners.

Integration of Uncertainty in Optimization Models

For problems where decisions are hence improved under uncertainty, as happens in (multi-objective) optimization problems, probability theory has gained wide acceptance as a powerful tool to represent uncertainties, which are summed up separately. For the same reason, in real-world applications, the straightforward application of such deterministic models often does not produce interesting or valid results as it trivializes the analysis. Overall, SO is based on a time-series approach, which comprises the use of past system performance (historical data) to predict uncertain factors for future operational planning purposes and is particularly fit to represent uncertainties that can be developed as random variables with given distributions [61, 62]. With (RO) models, uncertainties about the features of the constraints and the objective functions are addressed by using a set of possible (e.g., interval) values. Another approach that directly integrates forecasted values, and the associated errors in the uncertainty set to be used for the operational decision-making problems is the scenario approach. Scenario approaches have been referenced for various formulations, including SO, robust, or distributionally robust optimization [63, 64]. Recent years have witnessed a significant development of mathematical optimization techniques applied to support decision-making in sustainable energy systems engineering. A common factor in energy systems engineering problems is the intrinsic presence of uncertainty. For instance, due to the non-storability nature of electricity, the current electricity consumption should be matched with the current electricity generation, and errors in these predictions may lead to financial charges or even instability of power systems [65-67]. Other uncertain factors include load forecasts and electricity prices, fuel availability, and costs, renewable energy sources (RES) generation profiles, demand for products, market variables, and generation unit operational abilities. In the modeling context, this uncertain element has been mainly represented as forecasted values [68, 69], and optimization problems have been formulated as deterministic formulations. In this way, it is implicitly assumed that the forecasted values will be effectively realized, i.e., there is "no error" in the forecast. If such forecasts turn out to be inaccurate, the implemented model would need to be updated or replaced by more complex, and possibly nonlinear, uncertain representations to lead to solutions exhibiting satisfactory (even under uncertainty) performance. In the literature, two main ways to integrate uncertainty in optimization models have been discussed, including stochastic optimization (SO) and robust optimization (RO). By using SO, the uncertainty is represented via adopting a proper probability distribution function (PDF), whereas a fixed resource is more explicitly represented by the RO approach [71-73].

CONCLUSION

Advancements in mathematical optimization techniques are pivotal in tackling the complex challenges inherent in sustainable energy systems engineering. This paper has showcased both theoretical and practical advancements in optimizing key energy components, contributing to the development of efficient and resilient energy infrastructures. By leveraging multi-objective frameworks, these techniques facilitate robust decision-making processes for optimal investment, operational, and maintenance planning. The global transition to sustainable energy systems demands innovative solutions to manage the variability and uncertainty associated with renewable energy sources. Mathematical optimization provides a rigorous methodology for navigating these challenges, thereby enhancing the adoption and integration of sustainable energy technologies. Future research should focus on interdisciplinary approaches, combining domain-specific knowledge with advanced optimization techniques and AI-based modeling to further enhance the efficiency and effectiveness of sustainable energy systems. The collective advancements in mathematical optimization for sustainable energy systems engineering will significantly contribute to achieving global sustainability targets, ensuring a cleaner, more reliable, and resilient energy future.

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